Eur. Phys. J. C 5, 283–286 (1998)
DOI 10.1007/s100529800854

THE EUROPEAN
PHYSICAL JOURNAL C

Two-particle correlations and $B^0\overline{B}^0$ mixing

G.V. Dass[1], K.V.L. Sarma[2,*]

[1] Department of Physics, Indian Institute of Technology, Powai, Mumbai, 400 076 India (e-mail: guruv@niharika.phy.iitb.ernet.in, Fax: 091 22 578 3480)
[2] Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai, 400 005 India

Received: 24 November 1997 / Published online: 10 March 1998

Abstract. We study the EPR correlation implied by the entangled wavefunction of the $B^0\overline{B}^0$ pair created by the $\Upsilon(4S)$ resonance. The analysis uses the basis provided by the mass eigenstates B_1, B_2 rather than the flavour states $B^0, \overline{B}^0$. Data on the inclusive dilepton charge ratio are close to the expectation of quantum mechanics, but nearly 8 standard deviations away from that of complete decoherence. Our results are compared with those obtained in the $(B^0, \overline{B}^0)$ basis.

1 Introduction

Bertlmann and Grimus [1] have made an interesting use of the dilepton-decay data of the neutral bottom meson pair emitted by the $\Upsilon(4S)$. They focussed on a test of the two-particle correlation characteristic of quantum mechanics, as manifested over macroscopic distances ($\sim 10^{-3}$ cm). The tested feature of quantum mechanics is the consequence of interference between the two parts of the C-odd wavefunction

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}|B^0\overline{B}^0 - \overline{B}^0B^0\rangle, \tag{1}$$

where B^0 is the pseudoscalar bottom meson with $\bar{b}d$ as its valence quark constituents. The usual framework of the Weisskopf-Wigner approximation is assumed so that well-defined mixtures of the states B^0 and $\overline{B}^0$ propagate *independently* as B_1 and B_2 with masses (m_j) and inverse-lifetimes (Γ_j), $(j = 1, 2)$:

$$\begin{aligned} |B_1\rangle &= p|B^0\rangle + q|\overline{B}^0\rangle \\ |B_2\rangle &= p|B^0\rangle - q|\overline{B}^0\rangle. \end{aligned} \tag{2}$$

Here p and q are complex constants obeying the normalization condition $|p|^2 + |q|^2 = 1$, and invariance under CPT is assumed.

The analysis hinges mainly on two experimental ingredients: (i) the measured ratio of like-sign and unlike-sign dileptons in the chain decay

$$\Upsilon(4S) \to B^0\overline{B}^0 \to (\ell^{+,-} + \cdots) + (\ell^{+,-} + \cdots)$$

where ℓ stands for e or μ, and (ii) the mass difference $\Delta m = m_2 - m_1$ which is extracted from data on $B^0\overline{B}^0$ oscillations. The strategy adopted by Bertlmann and Grimus is to confront the data with the standard formula which is modified by introducing a factor $(1-\zeta)$ in the interference term. The results, as expected on the basis of other tests of quantum mechanics to date, are quite consistent with the prediction of quantum mechanics, i.e., with $\zeta = 0$. The value $\zeta = 1$, corresponding to the extreme case of adding probabilities for the time-dependent decays of the two parts $B^0\overline{B}^0$ and $\overline{B}^0B^0$ of the wavefunction of (1), is excluded. The $\Delta B = \Delta Q$ rule which provides a unique relationship between the lepton charge and the parent bottom flavour (as in the Standard Model) enters the analysis.

Correlation in the $(B^0, \overline{B}^0)$ basis is, however, different from that in the (B_1, B_2) basis. This is readily seen by expressing the initial state of (1) as

$$|\psi_0\rangle = \frac{1}{2\sqrt{2}pq}|B_2B_1 - B_1B_2\rangle, \tag{3}$$

and allowing the states (of definite momenta) to evolve to the respective decay instants.[1] The interference arising from decays of the parts $|B^0\overline{B}^0\rangle$ and $|\overline{B}^0B^0\rangle$ of (1) will not be the same as that from decays of the parts $|B_2B_1\rangle$ and $|B_1B_2\rangle$ of (3), in spite of the similar looking forms of (1) and (3). The reason is that a $|B_{1,2}\rangle$ remains a $|B_{1,2}\rangle$ on time-development, but a $|B^0\rangle$ picks up a $|\overline{B}^0\rangle$ component (similarly, $|\overline{B}^0\rangle$ picks up a $|B^0\rangle$ component) on time-development; the interference term in the (B_1, B_2) basis turns out to be comparatively simple. Independently of this simplicity, it is necessary to subject every correlation implied by quantum-mechanics to experimental checks;

[*] Died on 28 September 1997

[1] The assumption of CPT-invariance, whereby the constants p and q used in defining $|B_2\rangle$ become the same as those used in $|B_1\rangle$, is not necessary for the equivalence of the states given in (1) and (3)

the analysis in one basis is as important as that in any other. Analysis in the (B_1, B_2) basis is the motivation behind the present note. In the following, we multiply the interference between the decay amplitudes of the two parts of the state in (3) by a parameter E, so that $E = 1$ corresponds to the validity of quantum mechanics and $E = 0$ corresponds to the incoherent addition of decay probabilities. Obviously, E and $(1 - \zeta)$ are not identical because the interference terms they modify are very different.

2 Correlation in the two bases

We begin with the time-evolved state starting from (3)

$$|\psi(t',t)\rangle = \frac{1}{2\sqrt{2}pq} [\theta_2(t')\theta_1(t)|B_2B_1\rangle - \theta_1(t')\theta_2(t)|B_1B_2\rangle]. \quad (4)$$

Here t' is the proper time of the first beon (say, the one moving into the left hemisphere in the Υ frame), and t of the second beon; θ's are the evolution amplitudes

$$\theta_j(t) = \exp\left[\left(-im_j - \frac{\Gamma_j}{2}\right)t\right]; \quad (j = 1, 2).$$

The double-decay distribution for the first beon to decay into a channel denoted by β and the second to decay into another channel α is given (apart from an irrelevant overall constant) by

$$D[\beta(t'),\alpha(t)] = \frac{1}{8|pq|^2}|\theta_2(t')\theta_1(t)A_{2\beta}A_{1\alpha} - \theta_1(t')\theta_2(t)A_{1\beta}A_{2\alpha}|^2 \quad (5)$$

where

$$A_{1\alpha} = pA_\alpha + q\overline{A}_\alpha; \quad A_{2\alpha} = pA_\alpha - q\overline{A}_\alpha; \quad (6)$$

$$A_\alpha = \langle\alpha|T|B^0\rangle, \quad \overline{A}_\alpha = \langle\alpha|T|\overline{B}^0\rangle, \quad (7)$$

with similar definitions for the amplitudes corresponding to channel β.

We insert a time-independent, real parameter E (the 'EPR factor') in the interference between the two parts of the state in (3), so that $E = 1$ leads to the result of quantum mechanics. The case of $E = 0$ refers to complete decoherence, and is called the 'Furry hypothesis'. With the parameter E inserted, the time dependence reads as

$$D_E[\beta(t'),\alpha(t)] = \frac{1}{8|pq|^2}\{|\theta_2(t')\theta_1(t)A_{2\beta}A_{1\alpha}|^2 + |\theta_1(t')\theta_2(t)A_{1\beta}A_{2\alpha}|^2 - 2E\,\mathrm{Re}[\theta_2^*(t')\theta_1^*(t)A_{2\beta}^*A_{1\alpha}^*\theta_1(t')\theta_2(t)A_{1\beta}A_{2\alpha}]\}. (8)$$

In order to contrast this formula with the one obtained in the $(B^0, \overline{B}^0)$ basis [1], we list the "physical" states $|B^0(t)\rangle$ and $|\overline{B}^0(t)\rangle$ which evolve from the initial $|B^0\rangle$ and $|\overline{B}^0\rangle$ states:

$$|B^0(t)\rangle = \theta_+(t)|B^0\rangle + \frac{q}{p}\theta_-(t)|\overline{B}^0\rangle, \quad (9)$$

$$|\overline{B}^0(t)\rangle = \frac{p}{q}\theta_-(t)|B^0\rangle + \theta_+(t)|\overline{B}^0\rangle, \quad (10)$$

where

$$\theta_\pm(t) \equiv \tfrac{1}{2}[\theta_1(t) \pm \theta_2(t)].$$

The corresponding decay amplitudes which are time-dependent will be denoted by

$$\mathcal{A}_\alpha(t) \equiv \langle\alpha|T|B^0(t)\rangle = \theta_+(t)A_\alpha + \frac{q}{p}\theta_-(t)\overline{A}_\alpha, \quad (11)$$

$$\overline{\mathcal{A}}_\alpha(t) \equiv \langle\alpha|T|\overline{B}^0(t)\rangle = \frac{p}{q}\theta_-(t)A_\alpha + \theta_+(t)\overline{A}_\alpha, \quad (12)$$

and similar quantities for the channel β.

In the basis provided by B^0 and $\overline{B}^0$ therefore, the double-decay probability becomes

$$D[\beta(t'),\alpha(t)] = \tfrac{1}{2}|\mathcal{A}_\beta(t')\overline{\mathcal{A}}_\alpha(t) - \overline{\mathcal{A}}_\beta(t')\mathcal{A}_\alpha(t)|^2\,. \quad (13)$$

This expression is modified [1] by introducing the decoherence parameter ζ (following [2]) as

$$D_\zeta[\beta(t'),\alpha(t)] = \frac{1}{2}\{|\mathcal{A}_\beta(t')\overline{\mathcal{A}}_\alpha(t)|^2 + |\overline{\mathcal{A}}_\beta(t')\overline{\mathcal{A}}_\alpha(t)|^2 - 2(1-\zeta)\,\mathrm{Re}[\mathcal{A}_\beta^*(t')\overline{\mathcal{A}}_\alpha^*(t)\overline{\mathcal{A}}_\beta(t')\mathcal{A}_\alpha(t)]\}. (14)$$

We emphasize that while (5) and (13) are the same, their modified versions (8) and (14) are not. Hence there is no relation between the parameters E and ζ, except when quantum mechanics is valid which corresponds to $E = 1$ and $\zeta = 0$. It is also to be noted that in the extreme limit of complete decoherence corresponding to $E = 0$ and $\zeta = 1$, the two bases give different results.

Dileptonic charge ratio: To proceed further, we specialize the decay amplitudes of neutral beons to semileptonic channels which are labelled by m and $\overline{m}$, as follows:

$$A_m = \langle m_h\ell^+\nu_\ell|T|B^0\rangle, \quad (15)$$

$$\overline{A}_{\overline{m}} = \langle\overline{m}_h\ell^-\overline{\nu}_\ell|T|\overline{B}^0\rangle; \quad (16)$$

where m_h denotes a specified hadronic system having a net negative charge, and $\overline{m}_h$ denotes its CPT conjugate. In order to construct the fraction χ_d of inclusive like-sign dileptons, we first consider the decay rates into exclusive semileptonic channels, integrate the rates over t' and t, and sum over channel indices. In this way we obtain the leptonic combinations $\ell^+\ell^+$, $\ell^-\ell^-$, $(\ell^+\ell^- + \ell^-\ell^+)$. It is remarkable that even the inclusive time-integrated correlation among the lepton charges becomes useful [3] for testing the subtle effects of interference of a two-particle entangled wavefunction.

Using the formula (8), we express the dilepton ratio χ_d as

$$\chi_d \equiv \frac{R}{1+R} \quad (17)$$

$$R = \frac{N(\ell^+\ell^+) + N(\ell^-\ell^-)}{N(\ell^+\ell^-) + N(\ell^-\ell^+)} \quad (18)$$

$$= \frac{1-aE}{1+aE} \cdot \frac{|p/q|^2\sum_{m,n}|A_mA_n|^2 + |q/p|^2\sum_{\overline{m},\overline{n}}|\overline{A}_{\overline{m}}\overline{A}_{\overline{n}}|^2}{\sum_{m,\overline{n}}|A_m\overline{A}_{\overline{n}}|^2 + \sum_{\overline{m},n}|\overline{A}_{\overline{m}}A_n|^2}, \quad (19)$$

where for the sake of brevity we used

$$a \equiv \frac{1-y^2}{1+x^2}\,, \tag{20}$$

and the mixing parameters x and y are defined as usual:

$$x = \frac{m_2 - m_1}{\Gamma}, \quad y = \frac{\Gamma_2 - \Gamma_1}{2\Gamma}, \quad \Gamma = \frac{\Gamma_1 + \Gamma_2}{2}\,. \tag{21}$$

We now use a consequence of CPT invariance which states that the total semileptonic decay widths of the particle and its antiparticle are equal,

$$\sum_m |A_m|^2 = \sum_{\overline{n}} |\overline{A}_{\overline{n}}|^2,$$

provided we retain the weak Hamiltonian to first order and neglect the final state interactions due to electroweak forces. Using this result we obtain the formula for the likesign dilepton ratio,

$$\chi_d = \frac{u(1-aE)}{(1+u)+(1-u)aE}\,, \tag{22}$$

$$u \equiv \frac{1}{2}[|p/q|^2 + |q/p|^2]\,. \tag{23}$$

The parameter E can be determined

$$E = \frac{1+x^2}{1-y^2}\cdot\left[1 - \frac{2\chi_d}{u+(1-u)\chi_d}\right]\,, \tag{24}$$

by using the current experimental information on u, χ_d, x and y.

3 Experimental information

We note that it is adequate to replace the parameter u by unity in (24): The dilepton charge asymmetry $a_{\ell\ell}$ at the $\Upsilon(4S)$ measured by the CLEO group [4] is

$$a_{\ell\ell} \equiv \frac{N(\ell^+\ell^+) - N(\ell^-\ell^-)}{N(\ell^+\ell^+) + N(\ell^-\ell^-)} = (3.1 \pm 9.6 \pm 3.2)\times 10^{-2}. \tag{25}$$

As this asymmetry is expressible [5] in terms of the mixing parameters that define the mass eigenstates,

$$a_{\ell\ell} = \frac{|p|^4 - |q|^4}{|p|^4 + |q|^4}\,,$$

the experimental number implies that

$$u = \frac{1}{\sqrt{1-a_{\ell\ell}^2}} \simeq 1 + \frac{1}{2}a_{\ell\ell}^2 \tag{26}$$

$$\simeq 1.0005 \pm 0.031\,. \tag{27}$$

Consequently it is safe to use

$$u \simeq 1\,, \tag{28}$$

since the remaining parameters in (24) are known to much lower precision.

For the fraction χ_d, we take the average value given by the ARGUS [6] and CLEO [4] collaborations (see, the number called ‘our average’ on p. 506 of [7]),

$$\chi_d = 0.156 \pm 0.024\,. \tag{29}$$

This result is based on $\Upsilon(4S)$ data; it does not use the Δm from experiments on oscillations following Z decays.

For the value of x, we use the mass-difference Δm extracted from the time-dependence of $B_d^0\overline{B}_d^0$ oscillations in Z decays. It should however be emphasized that every oscillation experiment has backgrounds peculiar to it and their subtraction depends on the use of different algorithms. Background estimates and simulations do use the Standard Model, not to mention the framework provided by quantum mechanics; e.g., the jet-charge technique is based on modelling the b-quark fragmentation into a jet. Does it mean that in obtaining the experimental Δm, the theory we want to test is already used? The answer is “most probably not”: The oscillating B’s are created incoherently through the inclusive decays $Z \to B_d + \ldots$, while our focus is on a correlation relating to the coherence of a two-particle C-odd wavefunction.

Notwithstanding the above remark it is necessary to follow a conservative approach so that the results do not depend much on background estimates and related issues. For the present therefore, one may like to deal with data having a clean sample of identified B_d decays, as for instance in the data in which the semileptonic decays of B_d are reconstructed as completely as possible, although this would entail a considerable loss in statistics. The OPAL group [8] reported a sample of 1200 $D^{*\pm}\ell^{\mp}$ candidate events, of which 778 ± 84 were supposed to be arising from the decays $B_d \to D^*(2010)\ell\nu X$. The production flavour was determined by the jet charge method. Multiplying the Δm obtained in this experiment by the average lifetime of the B_d meson [7], we obtain

$$x_{\rm OPAL} = (0.539 \pm 0.060 \pm 0.024){\rm ps}^{-1}\cdot(1.56 \pm 0.06){\rm ps} \tag{30}$$

$$= 0.84 \pm 0.11, \tag{31}$$

where we have added the statistical and systematic errors in quadrature.

On substituting (28), (29), and (31) in (24), we obtain

$$E = \frac{1.17 \pm 0.15}{1-y^2}\,. \tag{32}$$

At present there is no direct experimental information on the parameter y. It is generally surmised that its magnitude cannot exceed a few percent mainly because there exists no flavourless channel into which both B_d and $\overline{B}_d^0$ decay dominantly. But fortunately, even values like $|y| \simeq 0.1$ hardly matter, as the correction to E is only quadratic in y. We may therefore set $y = 0$ to obtain

$$E = 1.17 \pm 0.15. \tag{33}$$

This value of E is consistent with $E = 1$ dictated by quantum mechanics, and is about eight standard deviations away from the Furry hypothesis ($E = 0$).

On the other hand, following [1], we might assume the validity of quantum mechanics ($E = 1$) and determine y from (32); the resulting bound is not restrictive: $|y| \leq 0.28$ at 90% confidence level.

To what value of ζ does our result in (33) correspond? For $y = 0$ and $u = 1$, we express the likesign dilepton fraction χ_d in terms of ζ to get

$$\zeta = \left[2\chi_d - \frac{x^2}{1+x^2}\right] \frac{(1+x^2)^2}{x^2} = -0.42 \pm 0.31\,. \quad (34)$$

The numerical value, which results from the substitution of (29) and (31), is comparable to the one standard-deviation limit $\zeta \leq 0.26$ of [1]. If we evaluate E from (24) using the data quoted in [1] ($\overline{x} = 0.74 \pm 0.05$ and $\overline{\chi}_d = 0.159 \pm 0.031$ along with $y = 0$ and $u = 1$), we would obtain $E = 1.06 \pm 0.11$.

4 Discussion

The use of $y = 0$ for getting the result in (33) is required by internal consistency since $y = 0$ is assumed in the determination of Δm which is used in (30).

The (B_1, B_2) basis turns out to have a greater sensitivity to possible modifications of the interference term as compared to the $(B^0, \overline{B}^0)$ basis. Using $y = 0$, the interference term in (8) with $E = 1$, is $[(x^2 + 1)/x^2]$ times that in (14) with $\zeta = 0$, if t and t' are fully integrated and if the decay channels considered are as required for the observable χ_d. Using (31) for x, this is a factor of (2.42 ± 0.36) in favour of the (B_1, B_2) basis.

The (B_1, B_2) basis has a slight edge over the $(B^0, \overline{B}^0)$ basis if possible violations of the $\Delta B = \Delta Q$ rule are allowed. If the amplitudes for these violations are retained to only the first order, the result in (22) is not modified for any E. In contrast, the corresponding result in the $(B^0, \overline{B}^0)$ basis *is* modified. However, these modifications must be proportional to ζ. The reason is that for $\zeta = 0$, (14) is identical to (8) for $E = 1$; there is then no modification at this matching point. Since ζ and possible violations of the $\Delta B = \Delta Q$ rule are both presumably small, this particular preference for the (B_1, B_2) basis is of only a limited importance.

In summary, data on the dileptonic decays of the $B_d^0\overline{B}_d^0$ pair belonging to $\Upsilon(4S)$ are analysed for possible violations of the two-particle correlation which occurs in quantum mechanics. This is done in the basis provided by the mass eigenstates $|B_1\rangle$ and $|B_2\rangle$. Because of the stronger interference term, this basis provides a greater sensitivity to possible modifications of the quantum-mechanical two particle correlations. Our result, (33), is completely consistent with quantum-mechanics, but disfavours the hypothesis of complete decoherence. Although our use of certain consequences of CPT invariance in B decays may be innocuous, the generality of our result is reduced by the use of the Weisskopf-Wigner approximation.

References

1. Bertlmann R.A., Grimus, W., Phys. Lett. B **392** (1997) 426
2. Eberhard P.H., in *The Second Daphne Physics Handbook*, Volume I, Miani L., Pancheri G., Paver N. (eds), (INFN, Frascati, 1995), p. 99
3. Datta A., Home, D., Phys. Lett. A **119** (1986) 3
4. CLEO Collab., Bartelt J., et al., Phys. Rev. Lett. **71** (1993) 1680
5. Okun L.B., Zakharov V.I., Pontecorvo B.M., Lett. Nuovo Cim. **13** (1975) 218
6. ARGUS Collab., Albrecht H., et al., Phys. Lett. B **324** (1994) 249; Z. Phys. C **55** (1992) 357
7. Particle Data Group, Barnett R.M., et al., Phys. Rev. D **54** (1996) 1
8. OPAL Collab., Alexander G., et al., Z. Phys. C **72** (1996) 377